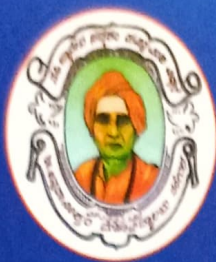


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**A PROJECT REPORT ON :**

**"APPLICATIONS  
ON LINEAR ALGEBRA"**

Submitted in partial fulfillment of requirements  
for the award of degree of

*Master of Science in  
Mathematics*

Submitted by

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6-8-2022

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## CHAPTER – I

### History of Linear Algebra

A procedure for solving Simultaneous Linear equations now called Gaussian Elimination appears in ancient Chinese mathematical text its use is illustrated in 18 problems, with two to five equations.

System of linear equation arose in Europe with the introduction in 1637 by Rene Descartes of Co-Ordinates geometry, It fact in this new geometry now called Cartesian geometry lines and plans are represented by linear – Equation and computing their intersections amounts to solving system of linear equations.

The first systematic methods for solving linear systems used determinants first considered by Leibniz in 1693. In 1750 Gabriel Cramer used them for giving explicit solutions of linear systems, now called Cramer's rule. Later Gauss further described the method of elimination. Which was initially listed as an advancement in geodesy.

In 1844 Hermann Grassmann Published his "Theory of Extension" which included foundational new topics of what is today called linear algebra in 1848 James Joseph Sylvester introduced the term matrix, which is latin for womb.

#### **UNIT:01 : INTRODUCTION**

First linear algebra is the study of certain algebra is the study of certain algebraic structure called a vector space, second linear algebra is the study of linear sets of linear sets of equations and their transformation properties. Finally it is the branch of mathematics charged with investigating the properties of finite dimensional vector space this project seek to give brief overview of history of linear algebra and its practical applications touching on the various topics used in concordance with it.

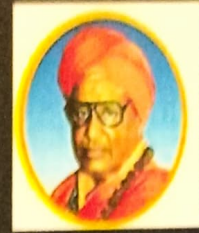
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## METHODS IN GAME THEORY AND IT'S APPLICATIONS

### HISTORY

The history of game theory is closely related to that of probability. The theory of probability developed because of gambling. Dice date back to at least 5,000 years ago. The dice below was found at Shahr-e-sukhteh (translated to Burnt city) in southeastern Iran in a back gammon set.

The foundation of mathematical probability can be traced back to 1654. Prominent gambler Antoine Gombaud was considering the following two lotteries. He could not figure out why he was winning on the first lottery and losing on the and one, and Huygene published his gambling calculus in *De ratiociniis in ludo aleae* [on resoning in games of chance] in 1657. Early approaches involved trying to find a winning strategy.

- ❖ Bouton dealt with Nim in 1901.
- ❖ Zermelo dealt with chess in 1913.
- ❖ Nash dealt with Hex in 1952.

### STRATEGIC INDEPENDENCE

Emil Borel wrote a series of papers between 1921 and 1927 where he set out to investigate whether it is possible to determine a method of play that is better than all others.

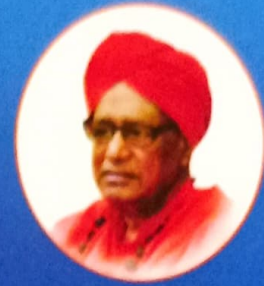


**Emil Borel**

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## CHAPTER I: FUZZY SETS

### HISTORY AND INTRODUCTION OF FUZZY SETS

#### 1.1. HISTORY:

The concept of fuzzy set was published in 1965 by Lotfi A. Zadeh. Since that seminal publication, the fuzzy set theory is widely studied and extended. Its application to control theory became successful and revolutionary especially in seventies and eighties, the application to data analysis, artificial intelligence, and computational intelligence are intensively developed, especially since nineties. The theory is also extended and generalized by means of the theories of triangular norms and conforms, and aggregation operators.

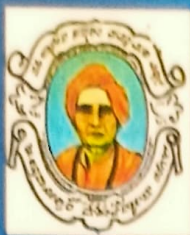


Lotfi A. Zadeh in 2004

A short historical review may be useful to better understand the character and motivation of this theory. The first publication in fuzzy set theory by Zadeh and Goguen show the intention of the authors to generalize the classical notation of a set and a proposition to

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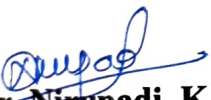



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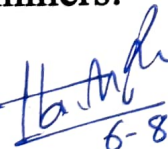
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CHAPTER – I  
BASIC TERMINOLGY  
**Metric Spaces**

**Introduction:**

With a strong understanding at analysis on the real line, the study of more general spaces & the relation limits & concept can begin. It is possible to generalize the fundamental concepts of real analysis in several different ways, but one of the most fruitful is in the context of **metric spaces**, where a **metric** is an **abstraction of a distance function**.

Our purpose in this brief discussion is not to develop the theory of metric space to any great extent, but to revival how the key ideas & technique of real analysis can be put into more abstract & general frame work.



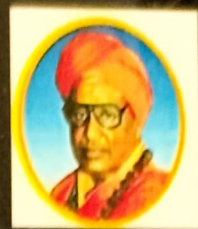
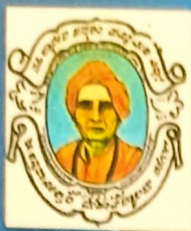
**Maurice Rene Frechet**

Maurice Rene (2 September 1878- 4<sup>th</sup> June 1973) was French mathematician. In 1906 Maurice introduced metric spaces in his work *Sur quelques points du calcul functional*.

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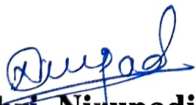
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06/08/2022

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## CHAPTER – I

### Introduction to Numerical Integration

#### **1.1 Introduction:**

Numerical Integration is the approximate computation of an integral using Numerical technique. The numerical computation of an integral is sometimes called quadrature. Quadrature is a historical mathematical term that means calculation of area.

Numerical Integration methods can generally be described as combining evaluation of the integrals to get or approximation to the integral is evaluated at a finite set of points is called integration points and a weighted sum of these values is used to approximate the integral. The integration points and weights depend on the specific method used and the accuracy required from the approximation.

An important part of the analysis of any numerical integration method is the study of behavior of the approximation over as a function of number of integral evaluation. A method that yields small error for a small number of evaluations is usually of arithmetic operations involved and reduces the total round off error. Also each evaluation takes time and the integrand may be arbitrary complicated.

#### **1.2 Origin of Numerical Integration:**

The first documented systematic technique capable of determining Integrals is the method of exhaustion of the ancient Greek astronomer Eudoxus, which sought to find areas & volume of by breaking them up into an infinite number of divisions for which the area or volume was known. This method was further developed & employed by Archimedes in the third century B. C. The next significant advances in integral calculus did not begin to appear until the 17<sup>th</sup> Century. At this time the work of cavalleri with his method of indivisibles work and by Fermat, began to lay the foundation of modern calculus, further steps were made in the 17<sup>th</sup> Century by Barrow and Torricelli, who provided the fast hints of a connection between integration and differentiation. .

The major advance in integration came in 17<sup>th</sup> century with the independent discovery of the fundamental theorem of calculus by Leibniz and Newton. The

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**DEPARTMENT OF STUDIES IN MATHEMATICS**

**A Project Report On  
"APPLICATIONS OF LAPLACE TRANSFORM IN  
INTEGRAL EQUATION"**

**Submitted in partial fulfillment of requirements  
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**Master of Science  
in  
Mathematics**

**Project Associate**

**Mr. Shrishail Hiremath**

**Reg. No. 20S14249**

**Under the Guidance of  
Miss. Nirmala Jutlad**

**2021-2022**

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## Chapter 1

### Laplace Transform

#### 1.1 History:

Transformation in mathematics deals with the conversion of one function to another function that may not be in the same domain. The transform method find its application in those problems which can't be solved directly. This transform is named after the mathematician and renowned astronomer Pierre Simon Laplace who lived in France.

He used a similar transform on his addition to the probability theory. It became popular after World War Two. This transform was made popular by *Oliver Haevisides*, an English electrical Engineer. Other famous scientists such as *Niels Abel*, *Mathias Lerch* and *Thomas Bromwich* used it in the 19<sup>th</sup> century.

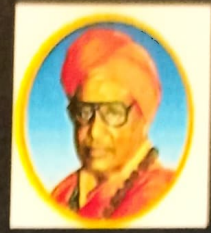
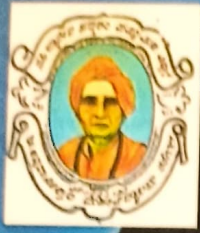
The complete history of the Laplace transforms can be tracked a little more to the past, more specifically 1744. This is when another great mathematician called *Leonhard Euler* was researching on other types of integrals. Euler however did not pursue it very far and left it. An admirer of Euler called *Joseph Lagrange*; made some modifications to Euler's work and did further work. Lagrange's work got Laplace's attention 38 years later, in 1782 where he continued to pick up where Euler left off. But it was not 3years later, in 1785 where Laplace had a stroke of genius and changed the way we solve differential equations forever. He continued to unlock the true power of the Laplace transform until 1809, where he started to use infinity as a integral condition.

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## DECLARATION

I hereby declare that, the project work reported in this dissertation entitled “**NUMERICAL LINEAR ALGEBRA**” is original work and is carried out by me under the guidance of **Miss. Madhu Beeranur**, Department of Mathematics, Shree Anandashwar Arts, Science, Commerce College & P.G. center Naregal. I further declare that this has not been previously presented for the award of any degree.

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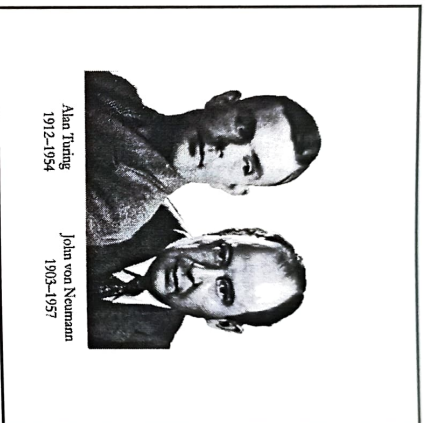


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## Chapter -1 Overview Of Numerical Linear Algebra

### 1.1 History of numerical linear algebra



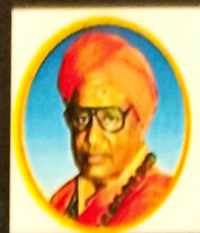
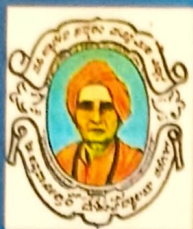
Numerical linear algebra was developed by computer pioneers like Jhon Von Neumann, Alan Turing, James. H.Walkinson, Alston Scott Householder, George forsythe and Heinz Rutishauser, in order to apply the earliest computers to problems in continuous mathematics , such as ballistic problems and the solution to the system of partial differential equations

The first serious attempt to minimize computer error in the application of algorithm to real data is Jhon Von Neumann and Hermann Goldstine's work in 1947. The field has grown as technology has increasingly enabled researchers solve complex problems as extremely large-high precision matrices and some numerical algorithms have grown in prominence as technologies like parallel. Computing have made them practical approaches to scientific problems.

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## **DECLARATION**

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## Chapter-1

### Basic Concepts of Differential Equation

#### History of Differential Equation

One of principal languages of science is that of differential equations. Interestingly the date of birth of differential equations is taken to be 11 November 1675, when Gottfried Wilhelm Freiherr Leibnitz (1646-1716) first put in black and white the identity  $\int y dy = \frac{1}{2}y^2$  there by introducing both the symbols  $\int$  and  $dy$ .

Leibnitz was actually interested in the problem of finding a curve whose tangents were prescribed. This led him to discover the “Method of Separation of Variables” in 1691. A year later he formulated the “Method of Solving the Homogeneous Differential of the First Order”. He went further in a very short time to the discovery of the “Method of Solving a Linear Differential Equation of the First Order”. How surprising is it that all these methods came from a single man and that too within 25 years of the birth of differential equation.

In the old days, what we now call the solution of a differential equation was used to be referred to as ‘Integral’ of the differential equation, the word being coined by James Bernoulli (1654-1705) in 1690. The word solution was first used by Joseph Louis Lagrange (1736-1813) in 1774, which was almost hundred years since the birth of differential equation. It was Jules Henri Poincare (1854-1912) who strongly advocated the use of the word ‘Solution’ and thus the word ‘Solution’ has found its deserved place in modern terminology. The name of the “Method of Separation of Variables” is due to John Bernoulli (1667-1748) a younger brother of James Bernoulli. John Bernoulli who first brought into light the intricate nature of differential equation.

**Conclusion:**

In this project I studied the methods of solving Simultaneous Differential Equation. The differential equation in which there is one independent variable and more than two dependent variables. To solve such equations completely, we shall require as many simultaneous equations as are the number of dependent variables. Simultaneous differential equation play major role in Applications of Science and engineering.

I introduce the methods of Total Differential Equation can be solved by four methods there are, Solution by Inspection method, One Variable as Constant method, Homogeneous equation, Auxiliary equation.

I can able to solve the simultaneous differential equations and apply to our knowledge and understanding to mathematics in everyday life.